UNIVERSITY OF
KWAZULU-NATAL

INYUVESI
YAKWAZULU-NATALI

SCHOOL OF ENGINEERING

MAIN EXAMINATIONS: JUNE 2023

COURSE AND CODE: FLUID AND SOLIDS TRANSPORT ENCH3FS

DURATION: 3 HOURS

TOTAL MARKS: 100

INTERNAL EXAMINERS: Dr. S. Moodley
Mr. C Baah

INTERNAL MODERATOR: Prof R. Rawatlal

EXTERNAL MODERATOR: Mr. T. Mungal

INSTRUCTIONS:

1. **ALL** questions should be attempted.

2. Any programmable or non-programmable calculator may be used provided it has been cleared of any information that would subvert the purpose of the examination.

3. Calculations must be shown in sufficient detail to illustrate your understanding of the procedure.

4. **Two answer books are provided. Label them Book A and Book B. Answer Section A in Book A and Section B in Book B.**

SECTION A

QUESTION 1

(a) From a survey of the literature on agitation, it has been established that the important variables are the rotational speed (N), impeller diameter (D), Tank diameter (TD), power input P, and fluid properties of density (ρ) and dynamic viscosity (μ). Although other variables may be important, perform a dimensional analysis for these six variables using Buckingham's π theorem.

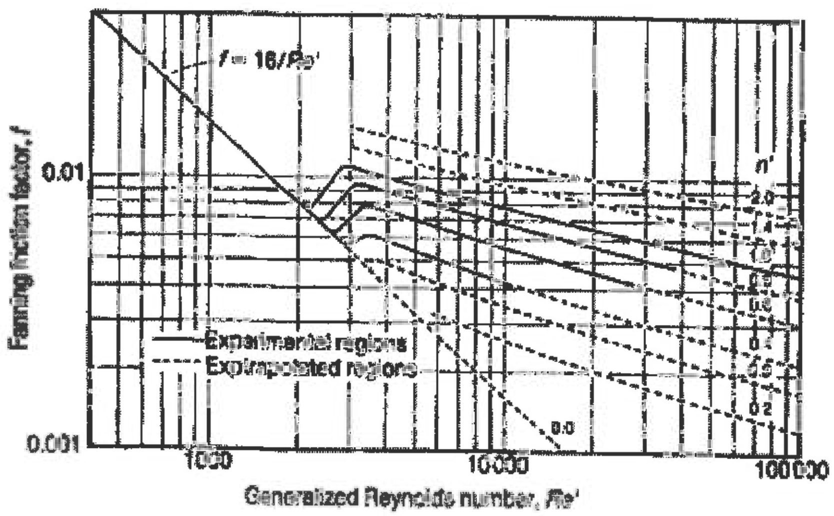
(16)

Symbol	Name	SI Units
N	Rotational speed	s^{-1}
D	Impeller diameter	m
TD	Tank diameter	m
P	Power	$kg.m^2.s^{-3}$
ρ	Density	$kg.m^{-3}$
μ	Dynamic viscosity	$kg.m^{-1}s^{-1}$

Use M, L,T dimensions and D, N and ρ as your recurring variables.

(b) A general time-independent non-Newtonian liquid of density 975 kg/m^3 flows steadily with an average velocity of 3.0 m/s through a tube 4.8 m long with an inside diameter of 0.091 m. For these conditions, the pipe flow consistency coefficient K has a value of 1.60 Pa n a value of 0.4. Calculate the pressure drop per meter length of the tube.

(17)



$$\mu_{ap} = K\left(\frac{8u}{d_i}\right)^{n-1}$$

TOTAL /33/

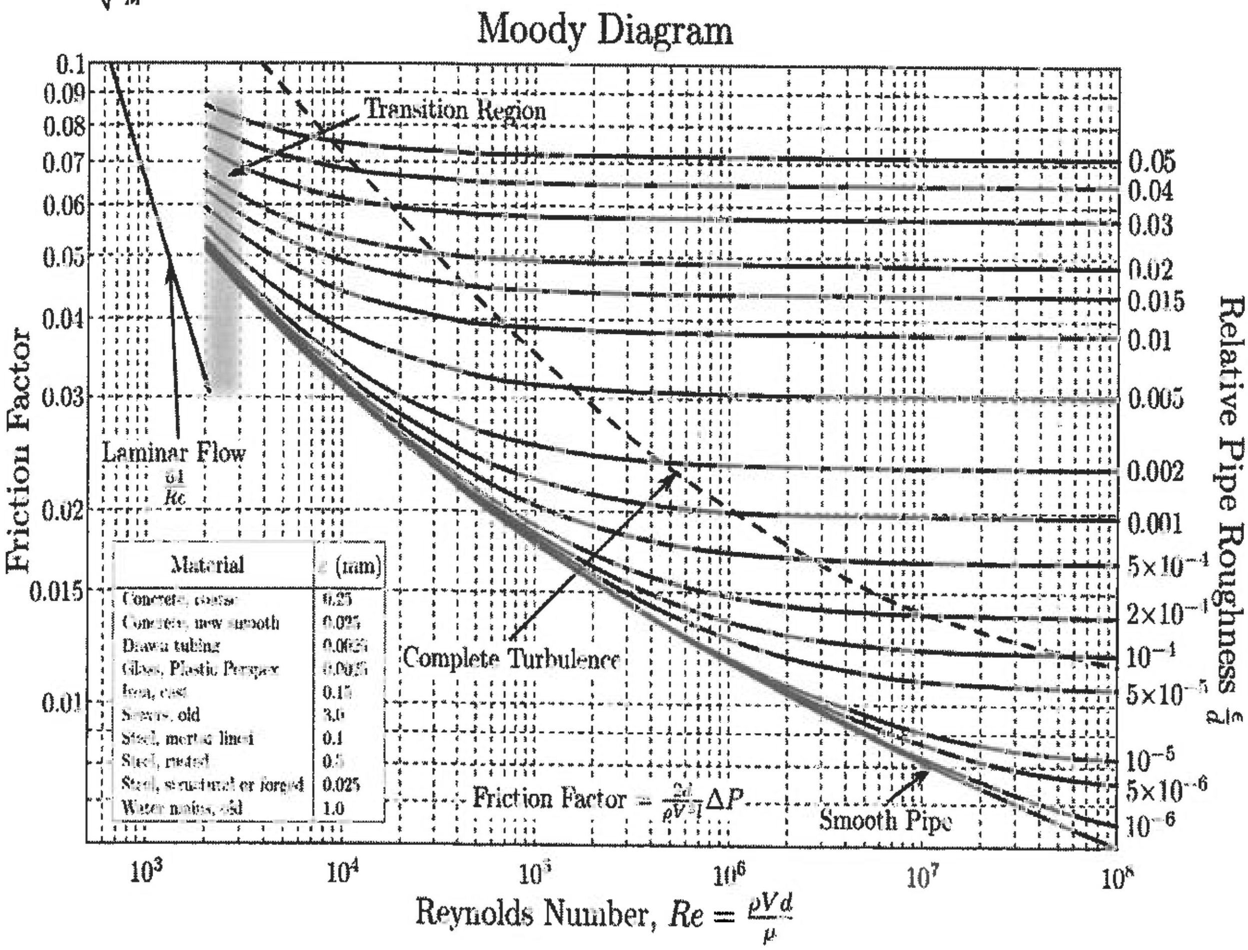
QUESTION 2

Ethane is to be transported through a steel (structural) pipeline of internal diameter 500 mm over a flat terrain to a chemical process plant. There are a couple of pumping stations along the pipeline. Each pumping station increases the pressure to 708 kPa (abs) and the pressure drops to 210 kPa (abs) at the inlet to the next pumping station 100 km away. The gas leaving the compressor cooler is 20°C (viscosity=1.05 x 10^{-5} Pa.s).
Estimate the flow rate in m^3/s measured at 20°C and 10^5 kPa. Assume isothermal and take $\frac{C_p}{C_v} = 1.31$

For isothermal:

$$1 - \left(\frac{P_2}{P_1}\right)^2 = 2\gamma Ma_{(1)}^2 \left[ln\frac{P_1}{P_2} + \frac{2fL}{d}\right]$$

$Ma_{(1)}^1 \frac{u_1}{2\sqrt{\frac{\gamma RT}{M}}}$ Where $Ma_{(1)}^1$ is the first iteration of Mach number at point (1)



TOTAL /17/

SECTION B

QUESTION 3

(a) In the lectures on fluidization, an ordinary differential equation (ODE) using Newton's second law was derived to describe the motion of a single, spherical particle of mass m as it fell under the influence of gravity in a viscous fluid. However, instead of solving the ODE to obtain an expression for the particle velocity u as a function of time t to eventually obtain an expression for the terminal velocity u_T of the particle, the du/dt term in the ODE was set to zero and the resulting equation simply rearranged to solve for u_T. However, Dr Moodley has found an analytical solution. The analytical (or exact) solution is

$$u(t) = \frac{\sqrt{a}\tanh\left[\sqrt{a}\sqrt{b}t + \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}u_0\right)\right]}{\sqrt{b}}$$

where

$$a = g\frac{(\rho_p - \rho_f)}{\rho_p}$$

and

$$b = \frac{C_d \rho_f A_p}{2m}$$

Note that u_0 is the initial velocity of the particle and A_p is the projected area of the sphere on a two-dimensional plane. Use the three preceding equations/expressions, and state all assumptions (if necessary), to show that the terminal velocity of a spherical particle of diameter d_p falling from rest is described by the following formula:

$$u_T = \sqrt{\frac{4g(\rho_p - \rho_f)d_p}{3C_d\rho_f}}$$

(8)

(b) Hence, calculate the terminal velocity of a spherical particle of diameter 1 mm (and density 3000 kg/m^3) falling in water (density 1000 kg/m^3 and viscosity 1×10^{-3} Pa · s). **Perform two iterations only**. A reasonable starting guess for the particle Reynolds number has an order of magnitude $\sim 10^2$.

(7)

(c) Hence, for a packed bed of diameter $D = 0.08$ m consisting of the particles from Part (b) above, calculate the mean bed voidage at a superficial velocity of 0.1 m/s.

(7)

QUESTION 3 (continued)

Supporting Information is provided on the next page.

Supporting Information:

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$
$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$
$$K = d_p\left[\frac{g\rho_f(\rho_p - \rho_f)}{\mu^2}\right]^{1/3}$$

When $K < 2.6$; $Re_p < 0.2$:	$C_d = 24/Re_p$
When $2.6 \le K \le 68.9$; $0.2 \le Re_p \le 500$:	$C_d = 24/Re_p + 3/\sqrt{Re_p} + 0.34$
When $68.9 < K < 2360$; $Re_p > 500$:	$C_d = 0.44$

$$\frac{4.8 - n}{n - 2.4} = 0.043Ar^{0.57}\left[1 - 1.24\left(\frac{d_p}{D}\right)^{0.27}\right]$$
$$u = u_t\varepsilon^n$$
$$Ar = \frac{gd_p^3\rho_f(\rho_p - \rho_f)}{\mu^2}$$

TOTAL /22/

QUESTION 4

A single spherical grain of an ore of true particle density $\rho_p = 2400$ kg/m^3 and diameter 1.2 mm, sinks in a liquid (density = 850 kg/m^3) with a terminal velocity of 0.4 m/s. In a suspension of ore particles in the liquid, the grain's hindered settling velocity is 0.1 m/s. In the second equation in the Supporting Information, $C = 0.415$.
Determine the viscosity (in Pa · s) of the pure liquid.

Supporting Information:

$$n = 4.6 - \mathrm{Re}\left[\frac{100 + 2.3\mathrm{Re}}{(1.5 + \mathrm{Re})(105 + \mathrm{Re})}\right]$$

$$u_\mathrm{H} = u_\mathrm{t}(1 - C)^n$$

$$\psi_\mathrm{P} = \frac{1}{10^{1.82(1-\varepsilon)}}$$

$$\mathrm{Re} = \frac{d_\mathrm{p}u_\mathrm{t}\rho_\mathrm{m}}{\mu_\mathrm{m}}$$

$$\rho_m = C\rho_p + (1 - C)\rho_f$$

QUESTION 4 (continued)

$$\mu_m = \frac{\mu}{\psi_p}$$

TOTAL /11/

QUESTION 5

During a variable-pressure-variable-rate filtration experiment using a plate-and-frame filter press, $10\ \mathrm{m}^3$ of filtrate was collected at a filtration rate $Q = 49.2\ \mathrm{m}^3/\mathrm{h}$. The filter press consisted of 30 frames of dimensions $1\ \mathrm{m} \times 1\ \mathrm{m} \times 0.015\ \mathrm{m}$. The filter medium resistance (R_m), assumed constant, was $7 \times 10^{10}\ \mathrm{m}^{-1}$.
The filter cake was compressible, and its resistance was found to obey the relationship

$$\alpha = 7 \times 10^9(-\Delta P_c)^s,$$

where the pressure drop has units of Pa and α has its usual SI units, while the pump characteristic was described by the formula

$$(-\Delta P) = -10000Q^2 + Q + 3,$$

where the pressure drop now has units of bar and Q has units of m^3/s. The amount of solids per unit volume of suspension (or slurry) was $9.96\ \mathrm{kg/m}^3$, and the filtrate viscosity was $1 \times 10^{-3}\ \mathrm{Pa \cdot s}$. The density of the solids was $2400\ \mathrm{kg/m}^3$. Determine the value of the cake compressibility parameter s by using the equation

$$\frac{\mu c_s V Q}{A^2} = \frac{(-\Delta P_c)^{1-s}}{\alpha_0(1-s)}$$

Hints:

- Note that a pressure conversion factor must be used for the given pump characteristic (overall pressure drop) equation.
- Derive an equation of the form $\Upsilon(s) = 0$ and solve the equation for s (i.e., find the root of Υ). **If an iterative computation is required**, then use the Newton-Raphson method (see Supporting Information) with a starting value of $s = 0.3$, and perform two (2) iterations.

Supporting Information:

$$x_{n+1} = x_n - \frac{f(x_n)}{\left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x_n}}$$

$$(-\Delta P_m) = \frac{\mu R_m Q}{A}$$

TOTAL /17/